

Quantitative Investing for the Individual Investor

by Geert Bekaert

Retirement investing for the individual

In the US, individual investors are channeling large fractions of their savings into the stock market, mostly through the purchasing of mutual fund shares. The mutual fund industry is very mature, and the performance of various mutual funds is closely monitored. Its stellar growth in recent years is partly due to the existence of so-called 401(k) plans. These plans are defined contribution plans for pensions that are increasingly being favored by companies over defined benefit plans. Individuals are allowed to save a fraction of their income tax-deferred with a cap of about \$10,000 per year. Most companies do not provide a standard pension plan, but supplement the savings of the individual, in some cases adding 2 percent of income to the savings of the individual for every 1 percent personally saved in the 401(k) plan, up to a certain maximum percent (for example, 10 percent of income). What is revolutionary about the scheme is that the individual herself decides how invest the money. Companies choose a number of mutual fund companies to provide a set of funds to invest in. The investment opportunities differ substantially across companies; in some plans, investors have only 10 funds from one mutual fund company to choose from, and in other plans there may be 150 different funds available from three different mutual fund companies. In both cases, the individual investor now faces a critical investment decision that may have important consequences for her well-being at retirement.

This investment decision is not only important, it is also daunting and difficult. Should she invest in stocks or bonds or both? What is the risk that she will not reach retirement goals, and how does this depend on the portfolio choice? What are the characteristics of the equity funds in the

plan: are they riskier than the market; do they invest primarily in value stocks or in small-cap high-growth stocks? How will the difference affect portfolio risk and return? Are the funds of good quality, or is the manager appropriating much of the returns through expenses and fees?

This development is not without interest for Europe. With the graying of the population, pay-as-you-go pension systems will necessarily have to be phased out or drastically adjusted. Private pension plans are likely to grow fast here, too, and are likely to be encouraged by cash-poor governments. I predict that, in the near future, most of the responsibility for providing a pension will shift to the individual, even in Europe.

Let us split up the investment problem into two distinct problems. The first is to find an appropriate asset allocation across cash, stocks and bonds. The second is to choose the right set of funds to reach that mix.

The asset-allocation problem

Some of the largest investors in the world, including many pension funds--in the US, at least, but also in the Netherlands--have resorted to sophisticated, quantitative techniques to guide the allocation of funds across different assets. They are being advised by consultants and give out mandates to sophisticated professional money managers who often use quantitative techniques to construct portfolios. Moreover, this asset-allocation problem is currently one of the hottest topics in financial economics research in the US, and the last few years have witnessed many Ph.D. students from top schools writing theses on this topic and landing jobs at other top schools. For example, in 1999, two of my own Ph.D. students at Stanford University became professors at Columbia University and UCLA, respectively, two top business schools in the US, and both of those students had written job-market papers on dynamic asset allocation.

Is there a scientific approach to the investment process? Can we learn something from all that research at the US's top research institutions? The answer is a resounding yes. Although we definitely have no well-accepted theory for how an investor should invest for the long run, some important lessons emerge from past and current research. I will briefly review some of the standard misconceptions on investments and offer the recent consensus opinion of the academic community. Out of this reflection follow a number of "golden investment rules." To provide some guidance on where I am headed, let me start by listing them here:

The Golden Investment Rules, Part I

- (1) Do not ignore stocks.
- (2) Diversify!
- (3) Diversify!
- (4) Diversify!
- (5) Do not time the market.
- (6) But do not buy and hold either.
- (7) Do not think stocks are only for the young.

(1) Do not ignore stocks

Stocks are riskier than bonds or cash investments, but that risk is rewarded. The equity premium, the additional return expected from stock investments over and above what a treasury bill investment earns, varies over time and across countries, but you can bet it is positive. In the US, it has historically been quite high, namely 6 percent. In most European countries, we lack the long time series the US has, and a conservative estimate of this premium would be 3 percent. Forgoing this extra return in a retirement portfolio is simply unwise. Only the very risk-averse should not hold stocks. This is not to say there are no risks attached to stock investments. Expected returns are not realized returns, and ex post some investors will be worse off than if they had invested in bonds, for example.

Nobody claims that you should put all of your money in stocks. How much is right for you depends on your risk aversion and how much wealth you hope to assemble for your retirement.

(2), (3) and (4) Diversify, diversify, and ... diversify

(a) Get rid of idiosyncratic risk

Maybe it is unwise to spread the gospel of stock investments too much, since a growing number of people are starting to become *very* stock-savvy. In the US, a number of people now try to make a living out of day trading, selling and buying stocks during the day; others use the Internet to track their stock portfolios and trade. In Europe, the equity culture seems to be taking off as well. The evolution is both encouraging and worrisome. Stock investing is wonderful if it is done right. In fact, there is little evidence in favor of the common investment practice of buying a few stocks. A good stock investment involves at least 40 to 50 stocks or more. This is based on some fundamental finance theory that really is indisputable and makes common sense, too. When you invest in an individual stock, you take on heaps of idiosyncratic risk that is not rewarded by higher expected returns: it is diversifiable. If this were not the case and you could create large portfolios with many securities, an arbitrage opportunity would present itself. Only common sources of risk are rewarded. This idiosyncratic risk can ruin anybody. Of course, you hear many stories of investors making a killing in this or that stock. However, do not forget that your friends will only tell you about their lucky buys and will undoubtedly fail to mention that less fortunate stock that disappeared from the stock market after a few years, leaving their money earning nothing. With a large stock portfolio, this idiosyncratic risk is diversified away, and only the so-called systematic risk remains. It is that risk that is rewarded and is found in large baskets of stocks: the market portfolio, small-cap stock index, an index of value securities. This

is the classic argument for volatility and risk reduction through diversification: do not put all your eggs in one basket. Mark Twain once said, paraphrasing the old saying, "Put all your money in one basket and watch that basket carefully." Mark Twain may have been an interesting writer, but he was not a finance guru. Stick to the old saying.

One problem with this principle of diversification is that it is not couched in terms that people care about. Volatility does not hurt when the stock market keeps going up, right? So if an individual stock has an expected return that is higher than the expected return on the market, should I not invest in the individual stock? What is not as well known is that individual stocks do have a chance to die (go bankrupt), and high-volatility stocks have a higher probability of disappearing than low-volatility stocks. Yes, volatility can eat your expected return and is in fact the main driver of the long-run distribution of returns. To illustrate this, I ran a simulation for the market portfolio and an individual stock over a 10-year horizon. Table 1 lists the main assumptions and presents some results, whereas Figures 1 and 2 graph the distribution of cumulative wealth. That is the value earned after 10 years, starting with \$1. The results are absolutely striking. Consider the results in Table 1. The average return on the market over 50,000 scenarios ends up being almost as high as on the stock. Having invested one dollar in the market for 10 years eventually yields 2.59 dollars when invested in the aggregate stock market but only 2.53 dollars when invested in the seemingly higher-yielding individual stock. Why? Well, in many realistic scenarios, our stock simply disappears; its value goes to zero. The median value, that is, the value that has a 50 percent probability of occurrence, is only 6 percent for the individual stock, a meager reward for all that risk taking. I also report 5 percent shortfalls, which record the cumulative wealth that 5 percent of scenarios fall below. This is a good measure of downside risk. For individual stocks, this is almost always zero, meaning that there is at least a 5 percent chance that

you will lose all your money when invested in an individual stock for such a long period. Is there any attraction to individual stock investing? Yes, they provide good lottery tickets. Turn to Figures 1 and 2. The distribution that we see in both cases, but much more pronounced for individual stocks, is skewed. That is, there is long right-hand-side tail of very positive returns, which are not very likely to occur. Yes, you do have a one in 100 chance that this particular individual stock investment will increase 21-fold in 10 years (see Table 1). It beats the lottery, where you have negative expected returns, since the lottery costs money to run and is a profit center for the government, but do you want to risk most of your wealth on the distribution in Figure 2?

Table 1

Return Simulations for the Market and for an Individual Stock

Assumptions

	Market	Stock
Expected return	10%	13%
Volatility	20%	45%
Investment horizon	10 years	10 years
Number of scenarios	50,000	50,000
Risk-free rate	5%	5%

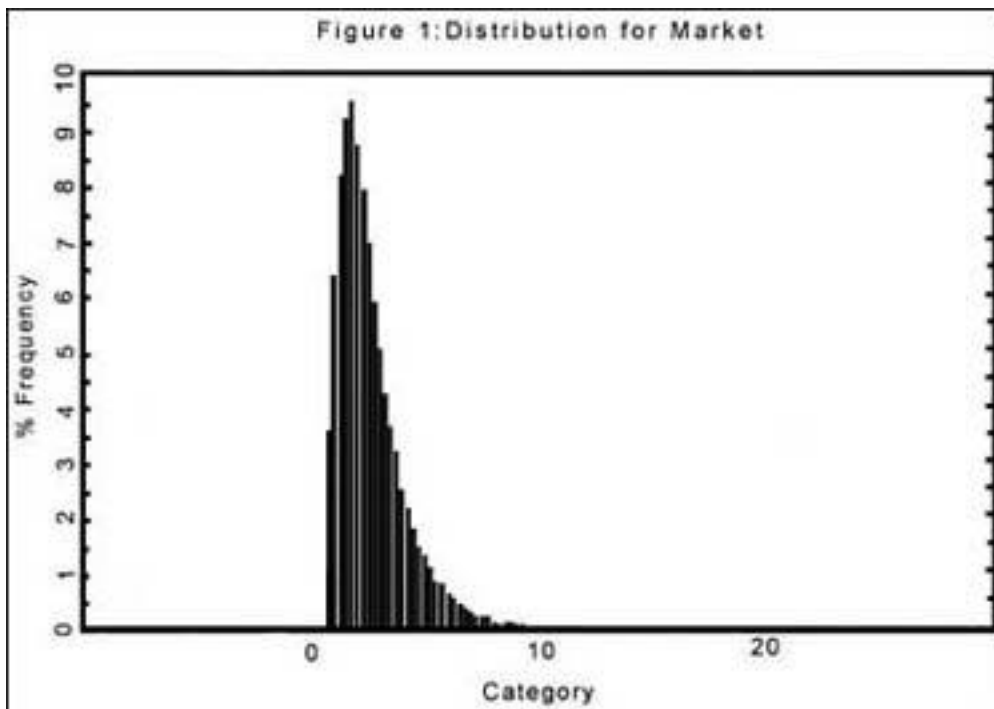
Notes: The market return is simulated as a log-normal random variable with mean and volatility such that the expected simple return is 10 percent and its volatility is 20 percent. The individual stock's net return follows the CAPM (capital asset pricing model), with a beta with respect to the market return of 1.6. The risk-free rate is mentioned in the table, and the innovation term is normal with volatility equal to 31.64 percent (the idiosyncratic volatility).

Results

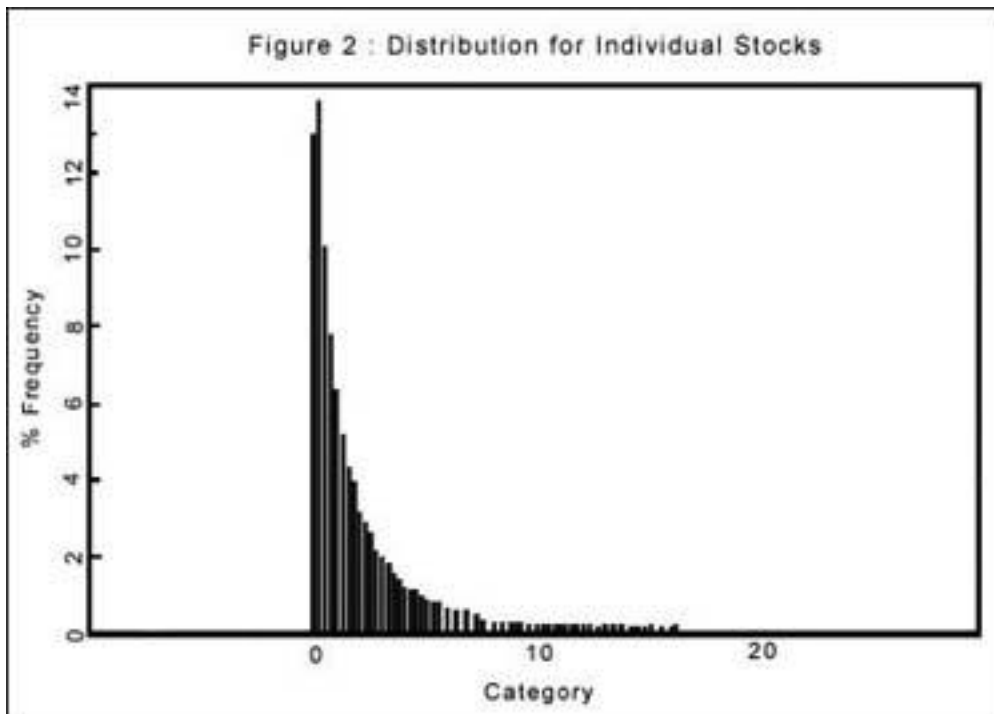
	Market	Stock
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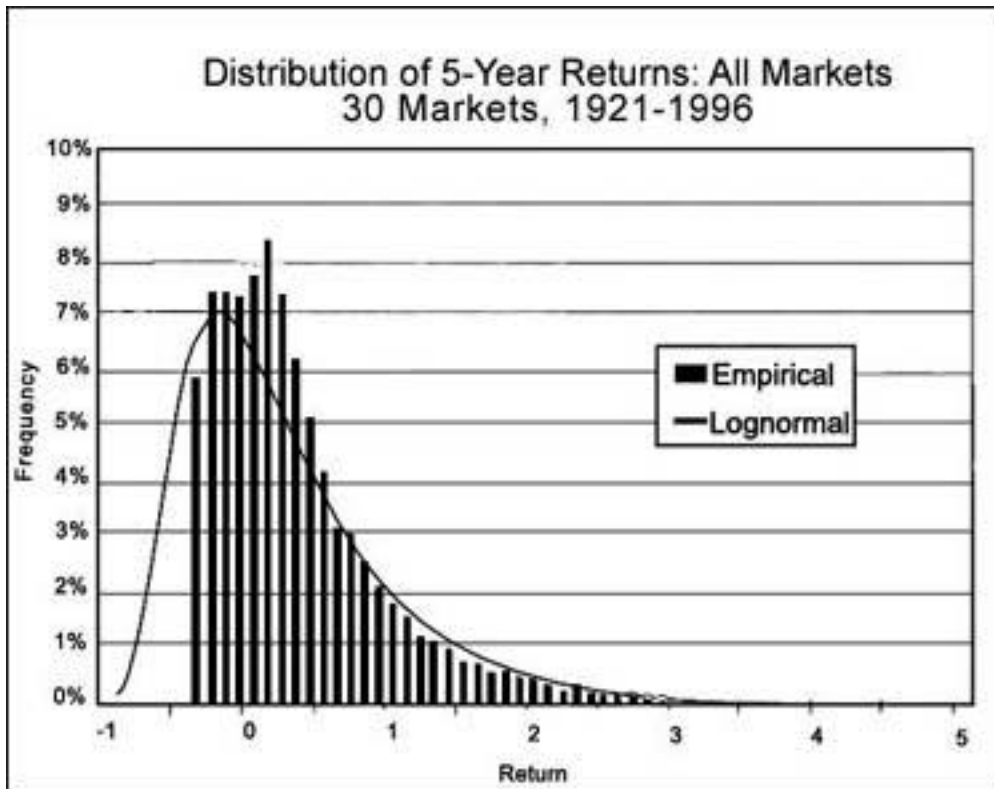
Average cumulative return	2.59	2.53
Deterministic return	2.59	3.39
Median return	2.20	1.06
Shortfall	0.86	0.00
Lottery wealth	8.37	21.14

Notes: When, in the simulation, the value of the market or stock reaches zero, the end value is set to zero. The average on the first line is taken over all 50,000 scenarios. The deterministic return is the cumulative return ignoring volatility. The median return is found by ordering all the different cumulative returns (50,000 in total) and taking the middle one (25,000th return). The shortfall similarly takes the 0.05 times 50,000 = 2,500th return. So only 5 percent of the scenarios yield cumulative returns that are lower than the shortfall. Hence, it is a measure of downside risk. The lottery wealth similarly takes the 0.99 times 50,000 = 49,500th return in the ranking: only 1 percent of the scenarios yield better cumulative returns.



Are my results robust? Two answers. First, at Financial Engines (a company in Silicon Valley, see below), we are actually testing models of volatility for individual stocks, and we are examining empirically how the long-run distribution of individual stocks behaves for US stocks. Figure 2 looks better than what we find! Second, consider Figure 3 from a recent paper by Philippe Jorion, which graphs an empirical distribution of all possible five-year returns (cumulative wealth minus one) from data on 30 stock markets (the returns are real and do not incorporate dividends). Note how the shape of the distribution resembles the shape of the distribution in our figures. Hence, the log-normal distribution assumption I made for the simulations seems to be borne out by the data.





(b) International diversification

Jorion's graph in interaction equity markets brings me to another important point. Do not restrict your basket too much. The eventual volatility of a portfolio of securities depends on the covariance (the correlation between the two portfolios times their volatilities) among the securities in it.

Adding low correlation assets really helps to reduce risk, and foreign securities help more than domestic ones. This is a fortiori true for any non-US country, since the US happens to be one of the least volatile markets in the world, perhaps because it is so large and well diversified itself.

Academics remain puzzled about why investors have such large fractions of their portfolios in domestic assets, and they have not been able to find a rational explanation for the phenomenon. The puzzle is known as the "home asset preference" or "home bias" puzzle.

A couple of caveats:

* It is often contended that the benefits of international correlation are

exaggerated because equity markets tend to be more highly correlated in bear markets, with the October 1987 crash as a prime example, during which most markets across the world went down. Whereas the empirical statement is undoubtedly true (see recent work by Francois Longin and Bruno Solnik at HEC), it does not mean that international diversification is a bust. In an article with Andrew Ang, we find that the presence of this particular time variation in correlation does not at all negate the benefits of international diversification. The periods of high correlation are too short and the correlation increases by too little to make much of a dent in the substantial benefits of international diversification.

* Of course, in Europe, correlations between the various markets are likely to increase as a logical consequence of the Single Market and the Single Currency. Diversification potential will have to be found elsewhere. Morgan Stanley Capital International follows some 20 developed equity markets, and the equity capitalization comprised by European Union countries is far less than 50 percent. But there are even better opportunities to diversify. The 1990s have witnessed a large number of capital-market liberalizations in developing economies, often coupled with rapid development of their securities markets and in particular their stock markets. Although well-publicized crises in Mexico (1994-5) and Southeast Asia (1997) have scared a lot of foreign investors, it is my opinion that these markets cannot be overlooked in any portfolio. From an economic perspective, it makes sense that mature developed economies help finance the development of these young economies, and one would suspect that there are more plentiful growth opportunities there. There is no doubt that emerging markets are risky investments when taken by themselves. The volatility of many of these markets often exceeds the volatility of well-established *individual* stocks at home. However, in a portfolio context, emerging markets are much less risky investments than, say, a high-tech, small-cap growth fund. The reason is their very low

correlations with developed market indices. Risk-averse to moderately risk-averse people should invest more in emerging markets than risk lovers invest: adding a little bit of emerging markets to your portfolio will reduce risk enormously. This is true even if you have doubts about expected returns in these markets. In my article with Mike Urias ("Is There a Free Lunch in Emerging Equity Investing?," *Journal of Portfolio Management*, Spring 1999), I conducted the following experiment. Suppose you could invest in a world market index of developed countries (using Morgan Stanley Capital International data) and an emerging market index. What is the additional expected return one needs on emerging market investments relative to the world index in order to make a 10 percent investment optimal? We tried several different indices, some easily accessible to retail investors (such as an index of open-end funds). The answer may surprise many, but it turns out it is negative: even if you believe expected returns in emerging markets to be lower than expected returns in developed markets (which in my opinion is hard to believe), you should still hold them in your portfolio. That is why I made investing in emerging markets my third golden diversification rule. Besides, after the current crises, prices are relatively low, although that partially reflects higher risk, including the risk that some markets may close to foreign investors or remain closed, as in Malaysia, which imposed capital controls in the aftermath of the Southeast Asian crisis.

(5) To time or not to time?

There is increasing evidence that certain macro and financial variables have predictive power for stock market returns. It is this evidence that makes quantitative asset managers salivate at the prospect of significant outperformance of the market. Nevertheless, how this evidence should affect optimal asset allocation is far from clear, and is the topic of very active research (see also above). It is easy to see the value in active timing. For example, suppose a particularly shrewd investor invests in the equity

markets of the G-5 countries (US, UK, France, Germany and Japan). Every month he attempts to put all of his money in the best-performing market. If he is right 50 percent of the time, one dollar invested on January 1, 1980, would be worth 1,414.58 dollars at the end of January 1999, before transaction costs.

It is also easy to see how hard market timing may be in practice. Bond/stock-market timers, for example, have a hard time figuring out when exactly to leave the stock market and an even harder time deciding when to re-enter. Missing a few well-performing months can be very costly for long-run performance. When in doubt about the stock market, stay invested. It is remarkable that in professional money management, the practice has grown to be evaluated relative to a market-capitalization-weighted benchmark. Active managers time by deviating from the index, but it also means that they never make huge bets and leave a large stock market all together.

(6) Do not buy and hold

Our previous rule does not necessarily imply that a buy-and-hold portfolio is the way to go. As is well known, portfolios have to be regularly reconsidered to assess whether they are still in accordance with the risk profile of the investor. The main problem with a buy-and-hold strategy is that the riskiness of the portfolio changes because higher expected return assets make up an increasingly larger part of the portfolio. Careless investors may end up with a very risky, near all-equity portfolio just before retirement, when rebalancing is ignored. The higher the transaction costs are, the less frequently the investor should rebalance. If the government were to tax capital gains, as is the case in the US, rebalancing could be very costly indeed. Ideally, the investor should figure out the optimal rebalancing frequency given transaction costs and taxes, but the current research technology does not allow solving such problems of

realistic dimensions. I suspect that, with realistic transaction costs, quarterly or even annual rebalancing is probably fine.

(7) Stocks are not only for the young

Financial planners in the US and elsewhere seem to link the proportion of investing in stocks relative to bonds to your time horizon. Simple rules such as the stock proportion equals 100 minus investor age are common. The practitioners' motivation for this rule is "time diversification." Stocks are less risky over the long run, they claim, hence people with longer horizons should hold more of them. There is surprisingly little academic evidence to back this statement up, and under standard assumptions about asset returns it is even patently false.

What is the source of the misconception? Well, stocks have high expected returns, but lots of short-term risk. If you cumulate historical returns, you indeed find that over longer and longer horizons, as Jeremy Siegel did in his often quoted work (*Stocks for the Long Run*, McGraw-Hill, 1998), the chances of having a negative return indeed decrease. There is no mystery about this, it is simple statistics, you are cumulating a random variable with a positive mean and lots of volatility. Does that mean the long-run stock investment is less risky? Of course not. The chance of losing may be less, but the *amount* you will be losing in bad times may be enormous. You cannot get rid of volatility in the long run. To put it very starkly, consider the pricing of options. Options allow agents to protect themselves to downside risk. That is, when you buy a put option on a stock at a certain strike price, you are allowed to sell that stock at the strike price. In other words, a stock investment coupled with a put option struck at the current price implies that you cannot lose more than the price of the option. When the stock goes up in value, you throw away the option; when the stock falls in value, you exercise your right to sell at the strike price. Since options allow you to protect yourself against downside risk, their prices

are not surprisingly an increasing function of volatility. If the Siegel argument is correct, surely long-term options on stocks must be cheap. In reality, the volatility of long-term stock investments rises with the horizon, and long-term options are very expensive.

What is the scientific evidence on the effect of horizon on the stock proportion? In fact, in the standard dynamic-asset allocation model going back to early work by Nobel Prize-winning economist Robert Merton, the stock-portfolio fraction is independent of age! This literature assumes stock returns are not predictable (prices follow random walks). With the more recent evidence on predictability (see above), the outcome is less clear but is dependent on the particular stochastic process that is assumed for returns. It may be the case that the financial planners end up being right after all, but not because of time diversification. At this time, however, I believe it is too soon to draw strong conclusions from this literature.

There are, however, two stories that potentially motivate holding more equity when young. The first is simple and well known among the better-informed practitioners. Stock returns may exhibit negative serial correlation at long horizons. What that means is that when stock returns have been unusually high for a while, one should expect them to decrease back to more normal levels. A model that would yield such behavior is one where stock prices tend to deviate from fundamental values, but only temporarily, and then move back to them. There is some statistical evidence in favor of this "mean reversion" hypothesis (see, for example, Jim Poterba and Larry Summers in a 1988 *Journal of Financial Economics* article), but it is in fact quite weak. If there is indeed mean reversion in stocks which is not present in bonds (for which there is also again weak statistical evidence), then the time diversifiers have a formal motivation for their rule. I remain skeptical.

A second story has to do with human capital, the complete set of your abilities that generates future income. For many young people, human capital constitutes the largest fraction of their wealth. Let's consider the example of the young university professor. In the beginning of his career, he likely has little investable wealth, but a substantial amount of earnings to look forward to, which, at least in Belgium, are very predictable, since they only depend on age (and not on merit and market demands). His human capital is the present value of all his future net earnings and should be counted as part of his wealth. In this case, the returns to human capital have a cashlike, at most bondlike, character, and the young professor ought to put most of his investable wealth in stocks, thereby hedging the relatively risk-free nature of his human capital. As he ages, the present value of future earnings goes down, investable wealth increases, and the human capital proportion falls. Hence, the proportion of investable wealth invested in stocks should decrease. Clearly, this story does not apply to everyone. The Wall Street equity trader's human capital may be highly correlated with the stock market, and he should then invest more and more of his investable wealth in stocks as he ages.

The Golden Investment Rules: Part II

Despite all the simple rules I have listed, it seems less straightforward to implement them in practice. I will present three simple rules regarding the practical implementation of a good investment plan:

- (1) Invest in mutual funds.
- (2) Choose "good" funds.
- (3) Choose efficient portfolios.

One immediate problem is that the rules I discussed before imply that investors should buy many individual stocks. Keeping track of such large portfolios and dealing with the dividend distributions, right issues, stock

splits, etc., seems inefficient for an individual. Moreover, fixed costs to trading securities imply that there are returns to scale. Enter mutual funds. They pool funds from many investors and make the investment process much less cumbersome. In the US, these funds are only a free phone call away, and they dispense tons of information regarding the investment opportunities they offer to investors. The mutual fund industry in the US is very mature and well developed. There are literally thousands of funds to choose from, and both private institutions (for example, Morningstar) and academics analyze the performance of these funds ad nauseam. A few years of bad performance can lead to a reduced inflow of funds and the slow death of the fund.

But what funds should one invest in, and how well do they perform? On the last question, the lessons of research are loud and clear. Many individual funds are not worth investing in! The return on a fund can best be analyzed using a simple regression procedure, developed by my former Nobel Prize-winning colleague at Stanford and the chairman of Financial Engines, William Sharpe, called style analysis. It splits up the past returns of the fund into a "style return" and "selection return." The style of a fund refers to its management style. For example, some managers offer funds investing only in medium maturity bonds; other managers have an equity portfolio, primarily invested in growth stocks. Clearly, the performance of the two managers cannot be compared, since they have different styles. Fortunately, some mutual funds offer so-called index funds. These funds passively invest in an index of securities, for instance, medium-term bonds, or growth stocks, or the S&P 500 in the US, or an index of securities in the Pacific (Japan, Australia, etc.). Since these funds do not require active management, they are cheap. The style return of a particular active fund is then the return on the portfolio of passive funds that best mimics the past return on the active fund, and the portfolio proportions represent the "style" of the fund (for example, it may be 80 percent small-

cap equity, 20 percent intermediate bonds). Hence, the style portfolio defines the style of the fund (e.g., growth equity manager versus value manager) and represents a portfolio that can be cheaply mimicked by investing in passive funds. However, the manager does *not* passively invest in the indices. He judiciously chooses particular securities, may buy certain stocks, sell others, etc. This active management is captured by the residual called selection return. Unfortunately, selection returns are often (after fees) negative. That means that the manager does not succeed in beating the passive indices, and that investors would have been better off investing in the much cheaper passive funds. Often funds that do outperform their style do not repeat that performance. Given that markets are relatively efficient, this is in fact not so surprising. There do exist superior investment skills. Peter Lynch from the legendary Magellan fund, for example, generated positive selection returns year after year. After he left the fund, performance went down, partly also because the increasing size of the fund had made it more difficult to beat the market.

How can you predict selection return? What are good funds? Well, we all wish we knew, and models to predict the performance of mutual funds are actively researched, but one relation is without question. Whereas in some product markets high prices are a signal of quality, this does not hold in the mutual fund market. Funds with high expense ratios, that is, high costs for the investor, have the worst performance--there is a very significant negative relation between expense ratios and mutual fund performance. Investors should especially beware of load funds. Charging a 3 percent load is theft. The chance that the manager will outperform the market consistently is very small. Although much of the research I am relying upon here is based on US mutual funds, I suspect that the situation is worse in Europe and particularly in Belgium. The numbers must be run first, but what worries me is the potential for some nasty clientele effects in Belgium. The mutual fund industry is not very mature, and, faced with

low interest rates, investors looking for higher yields get drawn into equity mutual funds by their savvy financial advisers, who most often happen to be in their main financial institution. This institution will of course only sell its own funds. If investors are not adequately informed about other possibilities, incentives to lower expenses and improve performance may be reduced relative to a cutthroat competitive marketplace such as in the US.

Suppose I finally get a list of "good" mutual funds together, and I have some vague idea of what wealth level I would like to reach at retirement, and some idea of the risks I am willing to incur in the process--which funds should I select? Sophisticated investors in the US, such as pension funds, approach this problem using quantitative techniques. They attempt to choose efficient portfolios. Ever since the pathbreaking work of Harry Markowitz (who won the Nobel Prize in 1990, together with William Sharpe and Merton Miller), it is well known that for every risk level there is a portfolio that yields the highest expected return, and vice versa. The portfolio weights for such portfolios can be obtained through a simple procedure called quadratic programming. Larger pension funds employ consultants to help them create such "efficient" portfolios and to simulate future scenarios of their assets so that they can stay on track with respect to liabilities, etc. The problem an individual faces is even harder. The horizon is quite long; he must forecast his income; he has tons of mutual funds to choose from, some of which may invest in similar securities without him knowing; etc. At present, there are no financial planners anywhere who have the technology or the knowledge to provide small individual investors with the kind of quantitative high-quality advice big institutional investors can rely on. With current computer power and the worldwide links of the Internet, it should now be possible to provide such advice even to individual investors at low cost. This is what a number of

start-up companies in the US are trying to do. Two of them are based in the San Francisco area (401K Forum, in San Francisco, and Financial Engines, in Palo Alto, in the heart of Silicon Valley). They both have the 401(k) market I talked about at the start of the article as a natural first target market, but through the Web, their advice could eventually become available for everyone in the world. I have some obvious preferences for one of those two companies, but that is another story.

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